Answer ALL TWENTY FOUR questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Simplify $a^7 \times a^4$

$a^{7+4} = a^{11}$

a^{11}

(1)

(b) Simplify $w^{15} \div w^3$

$w^{15-3} = w^{12}$

w^{12}

(1)

(c) Simplify $(8x^5y^3)^2$

$8^2 w^{5\times 2} y^{3\times 2}$

$64w^{10}y^6$

(2)

(d) Make t the subject of $c = t^3 - 8v$

$c + 8v = t^3$

$t = \sqrt[3]{c+8v}$

$t = \sqrt[3]{c+8v}$

(2)

(Total for Question 1 is 6 marks)

2 Danil, Gabriel and Hadley share some money in the ratios 3 : 5 : 9
The difference between the amount of money that Gabriel receives and the amount of money that Hadley receives is 196 euros.

Work out the amount of money that Danil receives.

D G H

3 5 9

$196 \div 4 = 49$

49×3

$= 147$

147 euros

(Total for Question 2 is 3 marks)

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3 The diagram shows triangle *ABC*

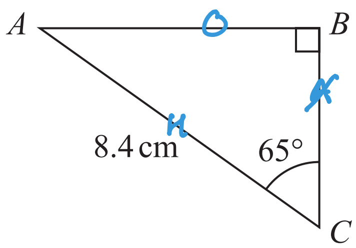

Diagram **NOT** accurately drawn

Work out the length of the side *AB*
Give your answer correct to 3 significant figures.

$\sin 65 = \frac{AB}{8.4}$

$AB = 8.4 \times \sin 65$

$= 7.61298$

(3sf)

7.61 cm

(Total for Question 3 is 3 marks)

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4 Sarah makes and sells mugs.

One day she makes 150 mugs.
Her total cost for making these mugs is £1140

Of these mugs

$\frac{2}{5}$ are small mugs

32% are medium mugs

and the rest are large mugs

Here is Sarah's price list for selling each mug.

MUGS	
Small	**£8.50**
Medium	**£11.20**
Large	**£14.20**

Sarah sells all 150 mugs.

Work out her percentage profit.
Give your answer correct to the nearest whole number.

150mugs = £1140

Small $\frac{2}{5} \times 150 = 60$ $60 \times 8.50 = 510$

Medium $0.32 \times 150 = 48$ $48 \times 11.20 = 537.60$

large $150 - (60 + 48) = 42$ $42 \times 14.20 = 596.40$

$+$ 1644

Profit = 1644 − 1140 = 504

$\% = \frac{504}{1140} \times 100 = 44.21...$

.......44.......%

(Total for Question 4 is 5 marks)

5 Jenny has six cards.

Each card has a whole number written on it so that

> the smallest number is 5
> the largest number is 24
> the median of the six numbers is 14
> the mode of the six numbers is 8

Jenny arranges her cards so that the numbers are in order of size.

5	8	8	20	21 or 22 or 23	24

6

14

4

(a) For the remaining four cards, write on each dotted line a number that could be on the card.

(3)

A basketball team plays 6 games.
After playing 5 games, the team has a mean score of 21 points per game.
After playing 6 games, the team has a mean score of 23 points per game.

(b) Work out the number of points the team scored in its 6th game.

5 games mean = 21 so total = 21 × 5 = 105

6 games mean = 23 so total = 23 × 6 = 138

Difference = 138 − 105
= 33

33

(3)

(Total for Question 5 is 6 marks)

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6 (a) Solve the inequality $5x - 7 \leqslant 2$

$5x \leqslant 9$

$x \leqslant \frac{9}{5}$

$x \leqslant 1.8$

(2)

(b) (i) Factorise $y^2 - 2y - 35$

1, 35
5, 7 ~~4~~ +5 −7

$(x + 5)(x - 7)$

$(x+5)(x-7)$

(2)

(ii) Hence, solve $y^2 - 2y - 35 = 0$

$(x + 5)(x - 7) = 0$

$-5 \quad 7$

$x = -5, x = 7$

(1)

(Total for Question 6 is 5 marks)

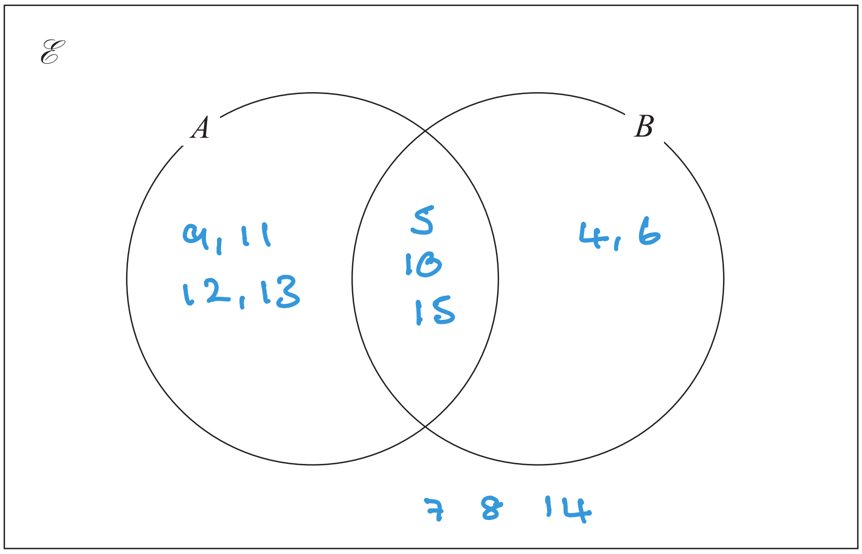
7 $\mathscr{E} = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$A \cap B = \{5, 10, 15\}$

$B' = \{7, 8, 9, 11, 12, 13, 14\}$ so $B = 4, 5, 6, 10, 15$

$A' = \{4, 6, 7, 8, 14\}$ so $A = 5, 9, 10, 11, 12, 13, 15$

Complete the Venn diagram for this information.



(Total for Question 7 is 3 marks)

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8 $a = 4.2 \times 10^{-24}$ $b = 3 \times 10^{145}$

Work out the value of $a \times b$
Give your answer in standard form.

$4.2 \times 3 \times 10^{-24} \times 10^{145}$

$12.6 \times 10^{-24+145}$

12.6×10^{121}

1.26×10^{122}

1.26×10^{122}

(Total for Question 8 is 2 marks)

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P 6 9 1 9 6 A 0 8 2 8

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9 The diagram shows isosceles triangle *ABC*

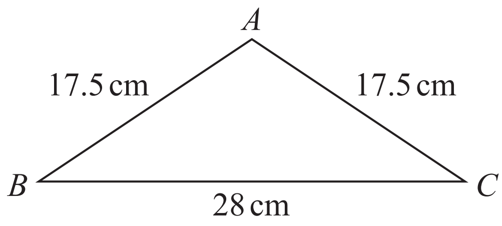

Diagram **NOT** accurately drawn

$AB = AC = 17.5$ cm $\qquad BC = 28$ cm

Calculate the area of triangle *ABC*

$a^2 = b^2 + c^2 - 2bc\cos A$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$= \dfrac{17.5^2 + 17.5^2 - 28^2}{2 \times 17.5 \times 17.5}$

$= -0.28$

$A = \cos^{-1}(-0.28)$

$= 106.260...$

$\text{Area} = \frac{1}{2}\, 17.5 \times 17.5 \times \sin 106.26...$

$= 147$

147 cm^2

(Total for Question 9 is 4 marks)

P 6 9 1 9 6 A 0 9 2 8

10 The straight line **L** has equation $2y + 7x = 10$

(a) Find the gradient of **L**

$2y = -7x + 10$

$y = -3.5x + 5$

-3.5

(2)

(b) Find the coordinates of the point where **L** crosses the y-axis.

$x = 0$

$y = -3.5 \times 0 + 5$

$= 5$

(0 , 5)

(1)

(Total for Question 10 is 3 marks)

P 6 9 1 9 6 A 0 1 0 2 8

11 Himari invests 200 000 yen for 3 years in a savings account paying compound interest.

The rate of interest is 1.8% for the first year and x% for each of the second year and the third year.

The value of the investment at the end of the third year is 209 754 yen.

Work out the value of x
Give your answer correct to one decimal place.

$200\,000 \times 1.018 \times x^2 = 209754$

$x^2 = \dfrac{209754}{200000 \times 1.018}$

$= 1.030..$

$x = \sqrt{1.030...}$

$= 1.01500...$

so 1.5%

$x =$ 1.5

(Total for Question 11 is 3 marks)

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12 The table gives information about the times, in minutes, taken by 80 customers to do their shopping in a supermarket.

Time taken (*t* minutes)	**Frequency**
$0 < t \leqslant 10$	7
$10 < t \leqslant 20$	26
$20 < t \leqslant 30$	24
$30 < t \leqslant 40$	14
$40 < t \leqslant 50$	7
$50 < t \leqslant 60$	2

(a) Complete the cumulative frequency table.

Time taken (*t* minutes)	**Cumulative frequency**
$0 < t \leqslant 10$	7
$0 < t \leqslant 20$	33
$0 < t \leqslant 30$	57
$0 < t \leqslant 40$	71
$0 < t \leqslant 50$	78
$0 < t \leqslant 60$	80

(1)

(b) On the grid opposite, draw a cumulative frequency graph for your table.

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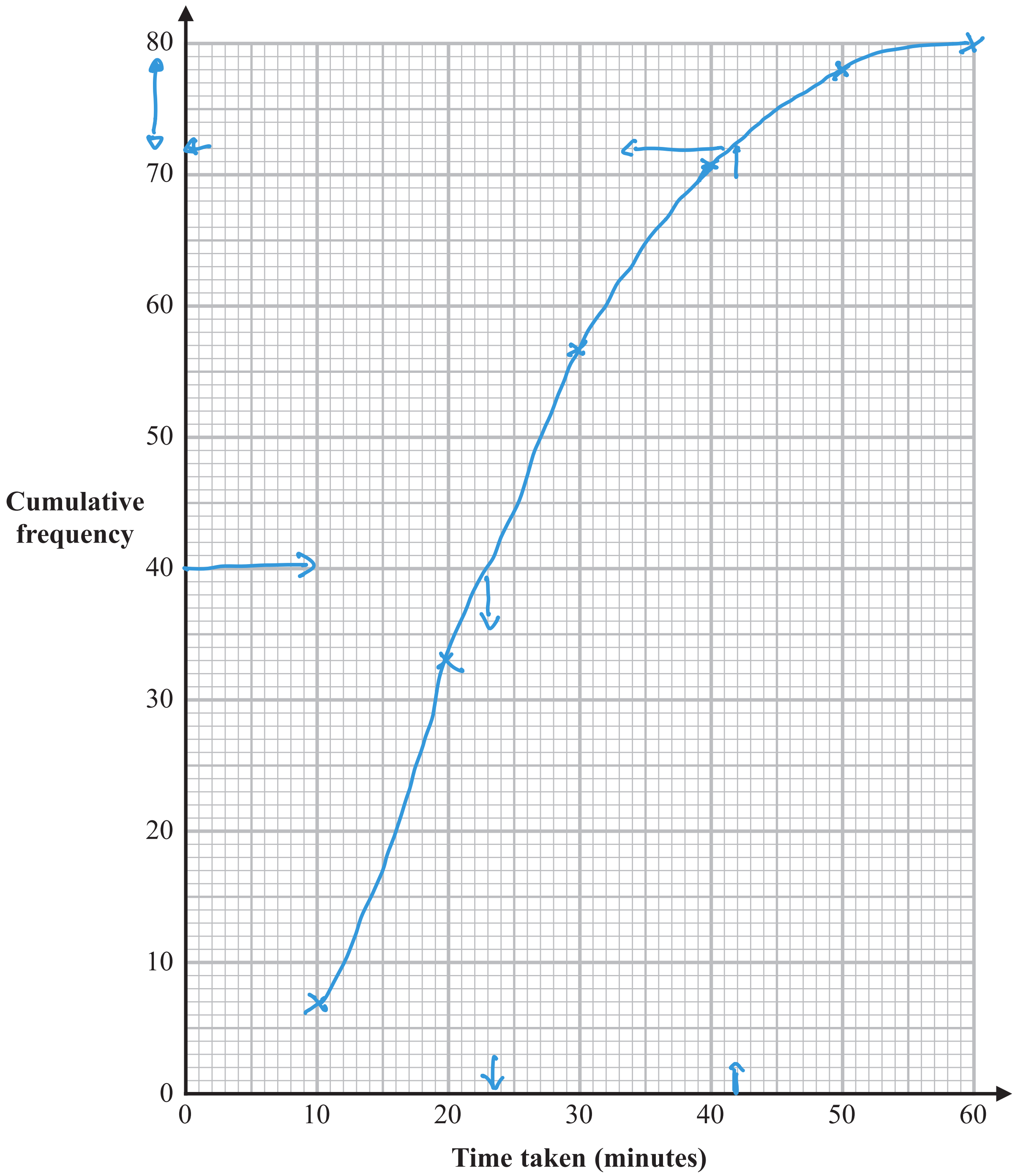

(2)

(c) Use your graph to find an estimate for the median time taken.

(range accepted of 21-24)

23 minutes

(1)

One of the 80 customers is chosen at random.

(d) Use your graph to find an estimate for the probability that the time taken by this customer was more than 42 minutes.

$\frac{8}{80}$

(2)

(Total for Question 12 is 6 marks)

13 (a) Expand and simplify $5x(x + 2)(3x - 4)$

$5x(3x^2 - 4x + 6x - 8)$

$= 15x^3 - 20x^2 + 30x^2 - 40x$

$= 15x^3 + 10x^2 - 40x$

$15x^3 + 10x^2 - 40x$

(3)

(b) Simplify completely $\left(\dfrac{16w^8}{y^{20}}\right)^{-\frac{3}{4}}$

$= \left(\dfrac{y^{20}}{16w^8}\right)^{\frac{3}{4}}$

$= \dfrac{y^{20 \times \frac{3}{4}}}{(\sqrt[4]{16})^3 \; w^{8 \times \frac{3}{4}}}$

$= \dfrac{y^{15}}{8w^6}$

$\dfrac{y^{15}}{8w^6}$

(3)

(Total for Question 13 is 6 marks)

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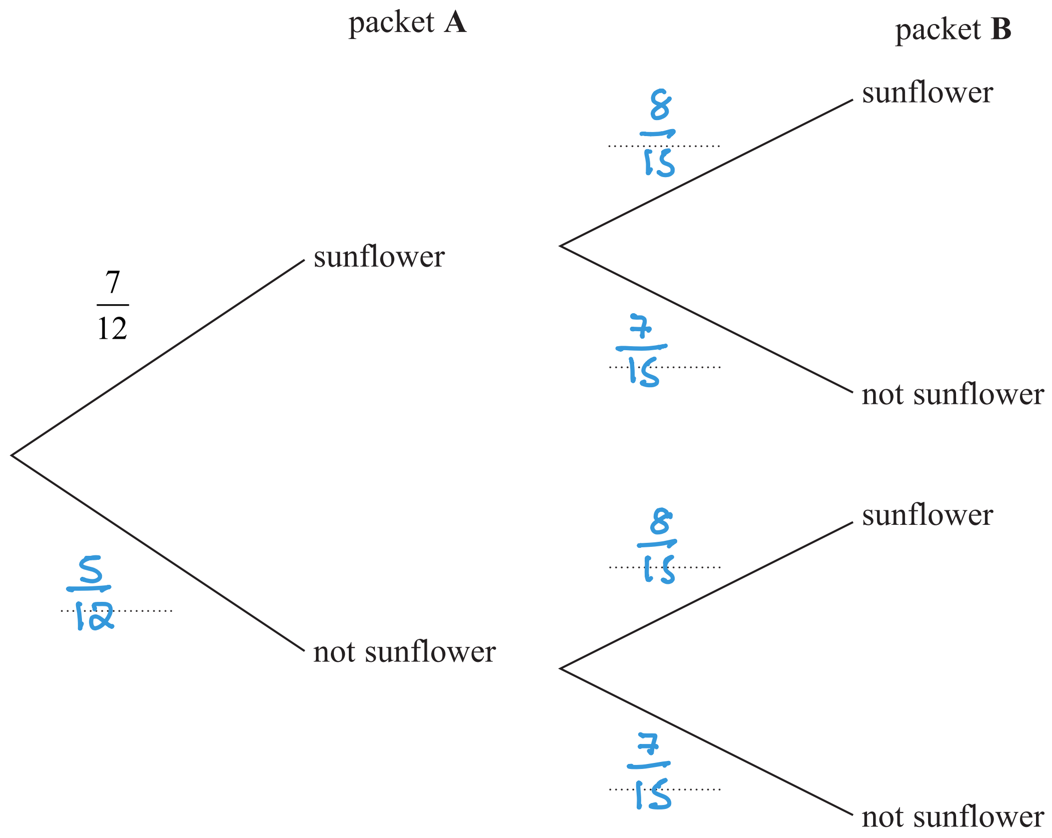
14 Aika has 2 packets of seeds, packet **A** and packet **B**

There are 12 seeds in packet **A** and 7 of these are sunflower seeds.
There are 15 seeds in packet **B** and 8 of these are sunflower seeds.

Aika is going to take at random a seed from packet **A** and a seed from packet **B**

(a) Complete the probability tree diagram.



(2)

(b) Calculate the probability that Aika will take two sunflower seeds.

$$\frac{7}{12} \times \frac{8}{15} = \frac{56}{180}$$

$\frac{56}{180}$ $\left(\frac{14}{45}\right)$

(2)

(Total for Question 14 is 4 marks)

15 A is inversely proportional to C^2

$A = 40$ when $C = 1.5$

Calculate the value of C when $A = 1000$

$A \propto \frac{1}{C^2}$ $A = \frac{k}{C^2}$ $40 = \frac{k}{1.5^2}$

$k = 40 \times 1.5^2$

$= 90$

$\therefore A = \frac{90}{C^2}$

$1000 = \frac{90}{C^2}$

$C^2 = \frac{9\cancel{0}}{100\cancel{0}}$

$= \sqrt{\frac{9}{100}} = \frac{3}{10}$

$C =$ $\frac{3}{10}$ (0.3)

(Total for Question 15 is 3 marks)

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P 6 9 1 9 6 A 0 1 6 2 8

16 The diagram shows a circle with centre O

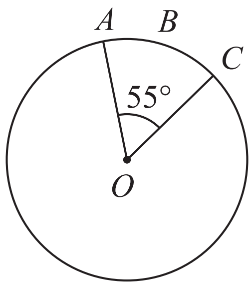

Diagram **NOT** accurately drawn

A, B and C are points on the circle so that the length of the arc ABC is 5 cm.

Given that angle $AOC = 55°$

work out the area of the circle.
Give your answer correct to one decimal place.

$$\frac{55}{360} \times \pi \times D = 5$$

$$D = \frac{360 \times 5}{55\pi} = 10.417\ldots$$

so radius = $5.2087\ldots$

$$\text{Area} = \pi \times 5.2087\ldots^2$$

$$= 85.23339\ldots$$

(1dp)

85.2 cm^2

(Total for Question 16 is 4 marks)

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17 **A** and **B** are two similar vases.

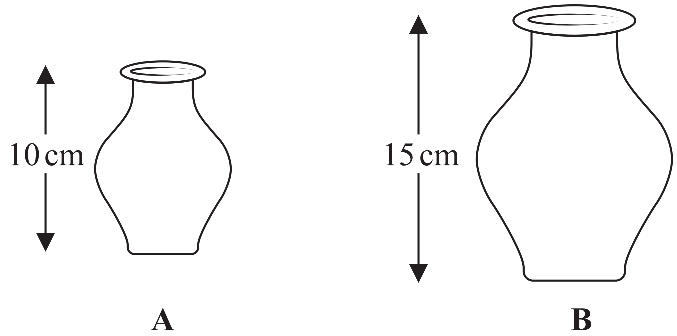

Diagram **NOT** accurately drawn

Vase **A** has height 10 cm.
Vase **B** has height 15 cm.

The difference between the volume of vase **A** and the volume of vase **B** is 1197 cm^3

Calculate the volume of vase **A**

length scale factor = 1.5

Volume scale factor = 1.5^3

= 3.375

Vol B - Vol A = 1197 cm^3

Vol B = Vol A × 3.375

3.375 Vol A - Vol A = 1197

Vol A (3.375 - 1) = 1197

Vol A = $\frac{1197}{2.375}$ = 504

504 cm^3

(Total for Question 17 is 4 marks)

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P 6 9 1 9 6 A 0 1 8 2 8

18 $A = w - \frac{x^2}{y}$

$w = 3.45$ correct to 2 decimal places.
$x = 1.9$ correct to 1 decimal place.
$y = 5$ correct to the nearest whole number.

Work out the lower bound of the value of A
Show your working clearly.

$w = 3.45$ ↙ 3.445 ↘ 3.455

$x = 1.9$ ↙ 1.85 ↘ 1.95

$y = 5$ ↙ 4.5 ↘ 5.5

$A_{LB} = 3.445 - \frac{1.95^2}{4.5}$

$A_{LB} = 2.6$

2.6

(Total for Question 18 is 3 marks)

19 Solve the simultaneous equations

$$3x^2 + y^2 - xy = 5$$
$$y = 2x - 3$$

Show clear algebraic working.

$$y^2 = (2x-3)(2x-3) = 4x^2 - 12x + 9$$

$$3x^2 + 4x^2 - 12x + 9 - x(2x-3) = 5$$

$$7x^2 - 12x + 4 - 2x^2 + 3x = 0$$

$$5x^2 - 9x + 4 = 0 \qquad 5 \times 4 = 20 \qquad 1, 20 \quad 2, 10 \quad 4, 5 \checkmark$$

$$5x^2 - 5x - 4x + 4 = 0$$

$$5x(x-1) - 4(x-1) = 0$$

$$(5x-4)(x-1) = 0$$

$$x = \frac{4}{5} \qquad x = 1$$

$$y = 2 \times \frac{4}{5} - 3 \qquad y = 2 \times 1 - 3$$

$$= -1.4 \qquad = -1$$

$x = 0.8$ $y = -1.4$, $x = 1$ $y = -1$

(Total for Question 19 is 5 marks)

20 (a) Express $7 + 12x - 3x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

$7 + 3(4x - x^2)$

$= 7 - 3(x^2 - 4x)$

$= 7 - [3(x - 2)^2 - 4]$

$= 7 - 3(x-2)^2 + 12$

$= 19 - 3(x-2)^2$

$19 - 3(x-2)^2$

(3)

C is the curve with equation $y = 7 + 12x - 3x^2$

The point A is the maximum point on **C**

(b) Use your answer to part (a) to write down the coordinates of A

(2 , 19)

(1)

(Total for Question 20 is 4 marks)

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P 6 9 1 9 6 A 0 2 1 2 8

21 The diagram shows the prism *ABCDEFGHJK* with horizontal base *AEFG*

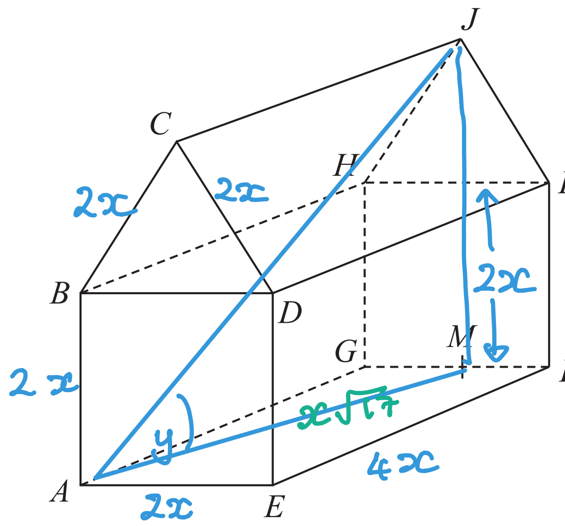


Diagram **NOT** accurately drawn

ABCDE is a cross section of the prism where
ABDE is a square
BCD is an equilateral triangle

$EF = 2 \times AE$
M is the midpoint of GF so that JM is vertical.

Angle $MAJ = y°$

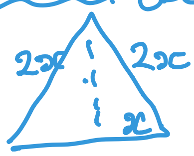
Given that $\tan y° = T$

find the value of T, giving your answer in the form $\dfrac{\sqrt{p} + \sqrt{q}}{17}$ where p and q are integers.

let $AB = BD = AE = DE = \cancel{x}$ $2x$ $\therefore$ $EF = \cancel{2x}$ $4x$

$AM = \sqrt{x^2 + (4x)^2} = \sqrt{17x^2} = x\sqrt{17}$

height of triangle

ΔJHK  $\sqrt{(2x)^2 - x^2} = \sqrt{4x^2 - x^2} = x\sqrt{3}$

$\tan y = \dfrac{2x + x\sqrt{3}}{x\sqrt{17}} = \dfrac{2 + \sqrt{3}}{\sqrt{17}}$

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$$\frac{\sqrt{p}+\sqrt{q}}{17}$$

so $\dfrac{2+\sqrt{3}}{\sqrt{17}} \times \dfrac{\sqrt{17}}{\sqrt{17}}$

$$= \frac{2\sqrt{17}+\sqrt{3}\sqrt{17}}{17}$$

$\sqrt{4}$

$$= \frac{2\sqrt{17}+\sqrt{51}}{17}$$

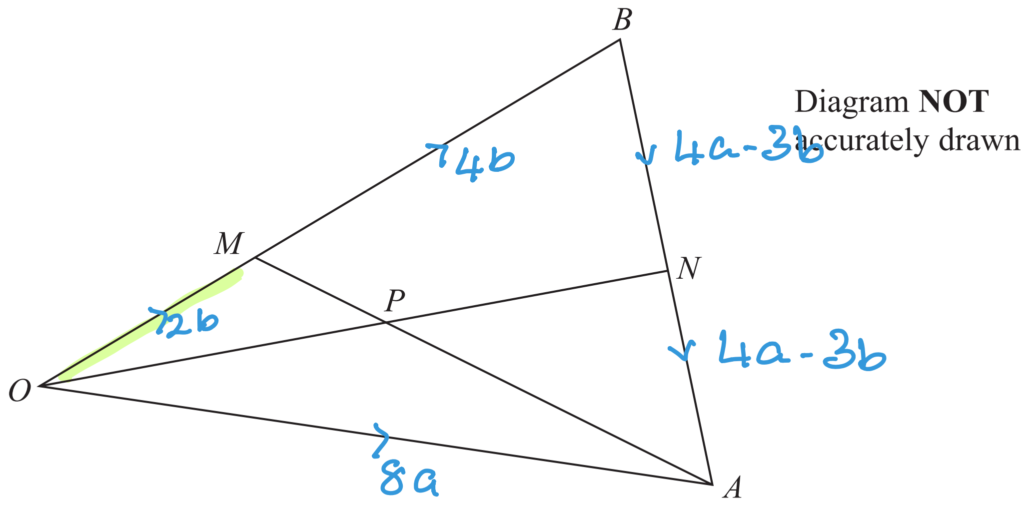
$$= \frac{\sqrt{68}+\sqrt{51}}{17}$$

$T = \dfrac{\sqrt{68}+\sqrt{51}}{17}$

(Total for Question 21 is 5 marks)

Turn over for Question 22

22 The diagram shows triangle OAB



$\overrightarrow{OA} = 8\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$

M is the point on OB such that $OM : MB = 1 : 2$
N is the midpoint of AB
P is the point of intersection of ON and AM

Using a vector method, find $\overrightarrow{OP}$ as a simplified expression in terms of **a** and **b**
Show your working clearly.

$OM = 2b$ $MB = 4b$

$BA = BO + OA = -6b + 8a$

$BN = NA = \frac{1}{2} BA = 4a - 3b$

$ON = 6b + 4a - 3b = 4a + 3b$

$MA = 8a - 2b$

$OP = \mu(4a + 3b)$ (using ON) $OP = 2b + \lambda(8a - 2b)$ using MA

P 6 9 1 9 6 A 0 2 4 2 8

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$OP = 4\mu a + 3\mu b$

$OP = 2b + 8\lambda a - 2\lambda b$

$= (2 - 2\lambda)b + 8\lambda a$

equating a and b

$4\mu = 8\lambda$ so $\mu = 2\lambda$

$3\mu = 2 - 2\lambda$

$3 \times 2\lambda = 2 - 2\lambda$

$6\lambda = 2 - 2\lambda$

$8\lambda = 2$

$\lambda = 0.25$

$\mu = 2 \times 0.25 = 0.5$

$OP = 4 \times 0.5a + 3 \times 0.5b$

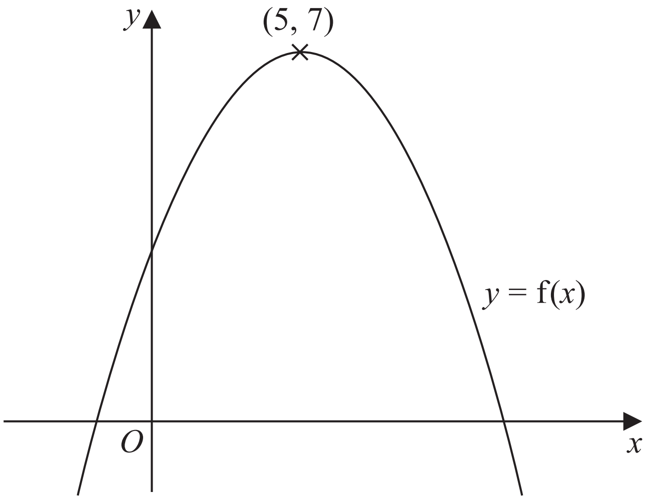
$= 2a + 1.5b$

$\overrightarrow{OP} =$ $2a + 1.5b$

(Total for Question 22 is 5 marks)

Turn over for Question 23

23 The diagram shows a sketch of the curve with equation $y = \mathrm{f}(x)$



There is only one maximum point on the curve.
The coordinates of this maximum point are (5, 7)

Write down the coordinates of the maximum point on the curve with equation

(i) $y = \mathrm{f}(x + 9)$

← 9

(-4 , 7)

(ii) $y = \mathrm{f}(x) + 3$

↑3

(5 , 10)

(Total for Question 23 is 2 marks)

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DO NOT WRITE IN THIS AREA

P 6 9 1 9 6 A 0 2 6 2 8

DO NOT WRITE IN THIS AREA

24 The curve **C** has equation $y = ax^3 + bx^2 - 12x + 6$ where a and b are constants.

The point A with coordinates (2, –6) lies on **C**
The gradient of the curve at A is 16

Find the y coordinate of the point on the curve whose x coordinate is 3
Show clear algebraic working.

at A $x = 2$ $y = -6$ $\qquad -6 = a \times 2^3 + b \times 2^2 - 12 \times 2 + 6$

$12 = 8a + 4b$

gradient at A

$\frac{dy}{dx} = 3ax^2 + 2bx - 12$

$16 = 3ax^2 + 2bx - 12$

$x = 2$ $\qquad 16 = 3 \times a \times 2^2 + 2b \times 2 - 12$

$16 = 12a + 4b - 12$

$28 = 12a + 4b$

so: $\quad 12 = 8a + 4b \quad (1)$

$28 = 12a + 4b \quad (2)$

$(2) - (1) \quad 16 = 4a$

$a = 4$

sub in (1) $\quad 12 = 8 \times 4 + 4b$

$12 - 32 = 4b$

$b = -\frac{20}{4}$

$= -5$

$\therefore y = 4x^3 - 5x^2 - 12x + 6$

when $x = 3$ $y = 33$

$y =$ 33

(Total for Question 24 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS

P 6 9 1 9 6 A 0 2 7 2 8